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# Berry's phase in a two-level atom

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Berry's phase in the coherent excitation of a two-level atom is shown to be an observable effect, as a shift of the sidebands of the Mollow's triplet. Berry's phase effects, the adiabatic following model, and nonadiabatic evolution of a two-level atom are discussed in a unified manner.

Berry's phase[1] has attracted a great deal of attention recently.[2] Berry's phase is a phase related to the phase factor of adiabatic theorem, known since 1928.[3] It has been taken to be an insignificant phase factor of quantum mechanics until recently when Berry, in a classic paper,[1] drew attention to the fact that the adiabatic phase has a circuit dependence, during a cyclic evolution, and that this phase should be observed. A two-level atom interacting with a coherent laser light, a well-known quantum optical system, is one of the potential candidates for the verification of Berry's phase. It is important to discuss Berry's phase in the two-level atom, first, because one of the simple examples chosen in the original paper[1] bears exact similarity to the dynamical equations of a two-level atom interacting with a coherent resonant laser light, and second, because the dynamics of the two-level atom has been studied in quantum optics literature fairly thoroughly over the last three decades. Then—why should Berry's phase not be observed in a quantum optics experiment? What experiment should be planned to see the effect? What do Berry's phase effects mean in resonance fluorescence in a two-level system whose dynamics has already been investigated using complete quantum electrodynamics? These are natural questions and have been addressed recently with interesting suggestions.[4] We show in the following that Berry's phase effects are measured numerous times, perhaps even in a single day, in almost all laboratories of quantum optics. The relationship between Berry's phase, adiabatic following model[5] of quantum optics, and nonadiabatic evolution is also discussed.

An important result of this paper is the demonstration that the quantum-mechanical phase factor of adiabatic theorem is measurable in the dynamics of a two-level atom interacting with a coherent laser light.

In order to demonstrate the above and in view of possible confusion later on, I prefer to begin *ab initio*. Adiabatic evolution and circuit in a parameter space are two ingredients of a discussion of Berry's phase, which we recall briefly in the following. Adiabatic evolution means that if the Hamiltonian changes slowly in time it is possible to approximate solution of the Schrödinger equation by means of eigenfunctions of the instantaneous Hamiltonian, so that a particular eigenfunction at one time goes over into the corresponding eigenfunction at a later time. The slowness of the variation of the Hamiltonian is determined by the ratio of the off-diagonal matrix element of the time derivative of the Hamiltonian, to the relevant energy separation between the eigenstates. That is, for the evolution to be adiabatic, one must have

$$\frac{\int(\bar{u}_k \partial H/\partial t u_n)d\tau}{E_k - E_n} \approx 0, \quad n \neq k\,, \tag{1}$$

where $u_n$ and $E_n$ are, respectively, the $n$th instantaneous eigenfunction and energy eigenvalue of the Hamiltonian, $H(t)$. For simplicity, the spectrum of eigenvalues is assumed to be discrete and characterized by the single subscript $n$. The evolution of the state function $\psi(t)$ of the system is then governed at a different time by

$$\psi_n(t) = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(t')\partial t'\right]\exp[i\gamma_n(t)]u_n\,. \tag{2}$$

The phase $\gamma_n(t)$ is the phase of the adiabatic theorem and is given by

$$\gamma_n(t) = \int_0^t (u_n \,|\, \partial/dt \,|\, u_n)\partial t\,. \tag{3}$$

Berry points out that $\gamma_n(t)$ does not necessarily return to its original value even when the Hamiltonian, having made an excursion in its parameter space, returns to its original value. The concept of the return to the original value of $H(t)$ defines a closed circuit in the parameter space of the Hamiltonian. The difference between the values of the phase $\gamma_n(t)$ before and after the completion of the circuit is referred to as Berry's phase.

Consider now a two-level atom, having eigenstates $|1\rangle$ and $|2\rangle$ with energy eigenvalues $E_1$ and $E_2$, respectively, interacting with a monochromatic radiation of angular frequency $\omega$ (with $\hbar\omega \approx E_1 - E_2$, $E_1 > E_2$). The Schrödinger equation of this system, in the slowly varying envelope approximation of radiation and the rotating wave approximation of the interaction, can be cast in the form[6]

$$i\hbar\,\partial_t \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \hbar\delta & -2d_{12}\epsilon \\ -2d_{12}^*\epsilon^* & -\hbar\delta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \quad (4)$$

$$d_{12} = \langle 1|ex|2\rangle,\ \hbar\delta = E_1 - E_2 - \hbar\omega, \quad (5)$$

$$(a,b) \rightarrow \exp\frac{i}{2\hbar}(E_1+E_2 \pm \hbar\omega)t(A,B), \quad (6)$$

$$|\psi(t)\rangle = A(t)\exp\left[-\frac{i}{\hbar}E_1 t\right]|1\rangle + B(t)\exp\left[-\frac{i}{\hbar}E_2 t\right]|2\rangle, \quad (7)$$

$$F(t) = \epsilon\exp(-i\omega t) + \epsilon^*\exp(+i\omega t), \quad (8)$$

with $F$ being the resonant electromagnetic radiation. In the familiar notation of a $2\times2$ Hamiltonian, one has $2H_{11} = -2H_{22} = \hbar\delta$ and $2H_{12} = H_{21}^* = X - iY$. We write in the following $X - iY = g\exp(i\alpha)$, where $g = -2|d_{12}\epsilon|$ and $\alpha$ are real. The eigenvalues of this $2\times2$ Hamiltonian are $\pm k$ with eigenstates $|\pm\rangle$,

$$4k^2 = (\hbar\delta)^2 + g^2. \quad (9)$$

The dynamics of Eq. (4), for the constant values of the parameters $\delta, g, \alpha$, is represented by the precession of the Bloch vector $V$ about a pseudomagnetic field $\mathbf{\Omega}$, defined by

$$V_x + iV_y = 2ab^*, \quad (10a)$$

$$V_z = |a|^2 - |b|^2, \quad (10b)$$

$$\mathbf{\Omega} = \left[\frac{g}{\hbar}\cos\alpha,\ \frac{g}{\hbar}\sin\alpha, \delta\right]. \quad (11)$$

The angular velocity of precession of $V$, which is in the clockwise direction, is $\Omega = |\mathbf{\Omega}|$. The angle between the vector $\mathbf{V}$ and $\mathbf{\Omega}$ is a constant of the motion and is determined by the initial conditions. Note that the precessing vector $\mathbf{V}$ represents a solution of Eq. (4) which is a superposition of the eigenstates $|\pm\rangle$. When the angle between $\mathbf{V}$ and $\mathbf{\Omega}$ is 0° or 180° there is no precession and the vector $\mathbf{V}$ remains fixed parallel or antiparallel to the vector $\mathbf{\Omega}$. The system is then said to be in the eigenstate $|+\rangle$ or $|-\rangle$ respective to the angle of 0° or 180° between $\mathbf{V}$ and $\mathbf{\Omega}$. It has recently been demonstrated[7] that it is possible to prepare a two-level atom in one of such eigenstates.

For the considerations of Berry's phase, we begin by assuming that the system has been prepared in one of the $|\pm\rangle$ states, say the $|-\rangle$. Note that the vector $\mathbf{\Omega}$ is a convenient representation for the variation of the parameters $\delta, g, \alpha$ of the $2\times2$ Hamiltonian. With the help of $\delta, g, \alpha$ the vector $\mathbf{\Omega}(t)$ can be varied in magnitude and inclination with respect to the Cartesian coordinate axes. One may model the variation of $\mathbf{\Omega}(t)$ by assuming[8]

$$\frac{d\mathbf{\Omega}}{dt} = \mathbf{\Omega}(t)\times\Xi(t) + \eta(t)\mathbf{\Omega}(t). \quad (12)$$

The first term in (12) represents the change in the inclination of $\mathbf{\Omega}(t)$, with respect to a vector $\Xi(t)$, and the second term represents the change in the magnitude of $\mathbf{\Omega}(t)$ by a scalar multiplicative factor $\eta(t)$. Consider a typical closed circuit in the parameter space when the tip of the vector $\mathbf{\Omega}(t)$ makes a circle about the $z$ axis, then $\Xi(t) = q\hat{z}$ and $\eta(t) = 0$. The vector $\mathbf{\Omega}(t)$ is then given by (11) with $\alpha$ replaced by

$$\alpha = \alpha_0 + qt. \quad (13)$$

Note that vector $\mathbf{\Omega}(t)$ returns to its original value after a period $T = 2\pi/q$. Note further that because of the finite return time the variation of the Hamiltonian may appear to be nonadiabatic; but as long as $q/k \approx 0$, the variation of $H$ can be taken to be adiabatic. The adiabatic evolution of the system, prepared initially in state, $|-\rangle$, $\phi_-(t_0) = 1$, and varied according to (13), is given by

$$|\psi(t)\rangle = \phi_+(t)|+\rangle + \phi_-(t)|-\rangle,$$

$$\phi_-(t) = \exp\left[\frac{i}{\hbar}k(t-t_0) + i\rho^2 q(t-t_0)\right]\phi_-(t_0), \quad (14)$$

$$\phi_+(t) = \phi_+(t_0) = 0.$$

Here $\gamma_-(t) = \rho^2 q(t-t_0)$ is the Berry's phase factor with $\rho = g/2[k(2k-\hbar\delta)]^{1/2}$. To examine the physical significance of this factor consider the two observables, namely, the coherent signal represented by (10a) and the number of photons emitted which is proportional to (10b). One finds for the solution (14)

$$V_x + iV_y = -2\zeta\rho\exp[i(\alpha_0+qt)]\,|\phi_-(t_0)|^2, \quad (15)$$

$$V_z = |\zeta|^2 - |\rho|^2, \quad (16)$$

$$\zeta = \frac{1}{2}\left[\frac{2k-\hbar\delta}{k}\right]^{1/2}. \quad (17)$$

Expressions (14) and (15) reveal two effects of the time-dependent Hamiltonian $\mathbf{\Omega}(t)$. First, it induces Berry's phase factor in (14); and second, it shifts the central frequency component of the coherent signal determined by (15).

The absence of Berry's phase factor in (15) is an expression of the fact that the phase factor of an eigenstate cannot be observed by examining the expectation values of the observables of the eigenstate. The situation is very different if one works with superposition of states $|\pm\rangle$. As noted above a superposition state represents for a constant Hamiltonian $\Omega_c = \Omega(\alpha = \alpha_0)$, the precession of the Bloch vector $V$ at a fixed angle to the vector $\Omega_c$. On the other hand, when the Hamiltonian $\Omega(t)$ is time dependent the superposition state can make complex trajectories of the tip of the vector $V$ on the Bloch sphere. Consider the cyclic Hamiltonian $\Omega(t) = \Omega(\alpha)$, $\alpha$ being defined by (13). The adiabatic evolution, due to this time-dependent Hamiltonian, of the superposition state represents precession of vector $V$ about the time-dependent vector $\Omega(t)$.[9] As a result, the expression similar to (15) demonstrates that the coherent signal contains three frequency components, viz.

$$\omega - q,\ \ \omega - 2q + 2\rho^2 q + \frac{2k}{\hbar},\ \text{and}\ \ \omega - 2\rho^2 q - \frac{2k}{\hbar}. \quad (18)$$

Comparing these components with the Mollow triplet[10] for the constant $\Omega_c = \Omega(\alpha = \alpha_0)$, one observes that for the $\Omega(t)$ defined by (13) above, the Berry's phase factor is contained in the shift of the sidebands. Thus Berry's phase factor is observable as frequency shifts of the sideband of the Mollow triplet spectrum.

Such effects are indeed observed as can be seen from the following *exact calculations*. Note that the time dependence (13) can be absorbed in the transformation (6) by redefining $\omega \rightarrow \omega - q$. Consequently, in (4) $\delta \rightarrow \delta + q$ and the problem of time-dependent Hamiltonian is reduced to a constant Hamiltonian for which the exact solution gives

$$V_x' + iV_y' = \zeta'\rho'(|\phi_+|^2 - |\phi_-|^2) - \rho' \exp\left[-\frac{2ik'}{\hbar}(t-t_0)\right]\phi_+\phi_-^* + \zeta'^2 \exp\left[+\frac{2ik'}{\hbar}(t-t_0)\right]\phi_-\phi_+^* . \quad (19)$$

$\zeta'$, $\rho'$, and $k'$ have the same expressions as before with $\delta$ replaced by $\delta + q$. The shift of the Rabi frequency due to a change of the laser frequency ($\omega \rightarrow \omega - q$) is clearly visible in (19), and it is too well known to be proven as an observable effect.

Note that for a general time-dependent Hamiltonian $\Omega(t)$, characterized by $\delta(t)$, $a(t)$, and $g(t)$, the Berry's phase factor for the $|\pm\rangle$ states of the two-level atom are given by

$$\gamma_\pm(t) = \mp \int_{t_0}^{t} \rho^2(t)\dot{a}(t')\partial t' , \quad (20)$$

and that the evolution of the atom either in the pure eigenstate $|\pm\rangle$ or in a superposition of them, can be considered adiabatic only when

$$\frac{[-i\hbar\rho^2(t)(d/dt)\zeta(t)/\rho(t) + \hbar\dot{a}\zeta(t)\rho(t)]\exp[-ia(t)]}{k(t) + |\rho^2(t)\dot{a}(t)|} \approx 0 . \quad (21)$$

Two important points are worth noting from (20) and (21). First, Berry's phase factors are nonzero only for $\dot{a} \neq 0$ (see also Barnett, Ellinas, and Dupertuis[4]), and second, adiabatic evolution in two-level atom dynamics can be achieved by making $k(t)$ much larger than the derivatives of $a$, $\delta$, and $g$.[11] This may be achieved in the experiments either by $g \gg (\dot{a},\dot{g},\dot{\delta})$ or $\delta \gg (\dot{a},\dot{g},\dot{\delta})$, or a combination of both. The first situation is implied in dressed atom theories,[12] and the second situation is considered in the adiabatic following model.[5] It may be noted here that Berry's phase effects were not considered in the adiabatic following model as only cases with $\dot{a} = 0$ were examined there.[13]

Clearly all shifts of the Rabi frequency sidebands are not a consequence of Berry's phase. Only such shifts of the sidebands which occur within adiabatic evolution and for constant $a$ and $g$ are comparable to Berry's phase effects and are not contaminated by other dynamical and nonadiabatic phase factors. Expression (19), for example, is valid in general and includes adiabatic, that is when $q \ll k \neq k'$, as well as nonadiabatic effects, i.e., when $q \gtrsim k \neq k'$. It has been shown that all corrections, due to higher-order iterations[14] to the results (18) are contained in (19).

We have thus shown that Berry's phase effects in coherent excitation of two-level atoms are observable. In view of the semiclassical nature of the discussion of Berry's phase, the question of its observability has been discussed here using the semiclassical observables.[15] The quantum-mechanical spectrum of the emitted photons can also be discussed; however, its dependence on the observables (15) and (16) is well known[10] and need not be discussed here. Nevertheless, it must be noted that *unlike* the semiclassical discussion above[16] (and also in Ref. 4), the *quantum-mechanical spectrum shows* the shift of the sidebands *even when* the atom has been prepared in one of the eigenstates, $|+\rangle$ or $|-\rangle$. It has been shown,[17] for example, that the shift appears during the transient regime, in the particular sideband present during the initial stages.[18] Such an effect cannot be discussed in semiclassical language because of the intrinsic quantum nature of the resonance fluorescence spectrum.

Frequency splitting or shift due to Berry's phase has been demonstrated earlier in nuclear quadrupole[19] and in optical polarization phenomenon in anisotropic crystals.[20] In these experiments either the quadrupole coupling sample or the anisotropic plate is rotated. In contrast, the two-level example of quantum optics is in exact spirit of the original suggestion[1] for a spin-$\frac{1}{2}$ Hamiltonian, wherein the (effective) magnetic field completes the cycle. It is an example for which the full quantum electrodynamical results are known. Thus, the semiclassical and the quantum electrodynamical "Berry's phase" effects can be contrasted—as pointed out above.[17]

In conclusion, we have demonstrated here that Berry's phase effects in coherent excitation of the two-level atom are observable by comparing the dynamics in two rotating frames of reference, which is what it must be, since all motion, be that of a Hamiltonian, $\Omega(t)$ are relative.

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